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Twin-correlations in atoms

K. Mølmer

Institute of Physics and Astronomy, University of Aarhus, 8000 Århus C, Denmark

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Abstract. We discuss the possibility of preparing an atomic sample of atoms with minimum fluctuations in the difference between populations of two levels. A first scheme involves absorption of twin beams of light, and it presents a variant of a recent proposal for atomic spin squeezing within an excited state manifold [Kuzmich *et al.*, Phys. Rev. Lett. **79**, 4782 (1997)]. A second scheme involves atoms with two stable states, and we suggest that by use of quantum non-demolition detection and feed-back optical pumping, we may ensure a perfect agreement between the number of atoms in these two states.

PACS. 42.50.Lc Quantum fluctuations, quantum noise, and quantum jumps – 03.70.+k Theory of quantized fields – 75.10.Jm Quantized spin models

1 Introduction

In precision spectroscopy on atomic samples there are various noise sources contributing to the total measurement uncertainty. The application of non-classical states of the electromagnetic field such as squeezed states and twin-beams has been shown [1] to reduce the effects of quantum noise of the field, and in a variety of experiments, the contributions of the quantum noise of the atoms have now been observed [2–5]. It has therefore been proposed to produce squeezed or quantum entangled states of the atomic system in order to reduce also the atomic noise contribution.

One line of research in this direction has dealt with spin squeezed states of ions, accommodated by the complete control of the state of such a multi-particle system due to the coupling of internal and external degrees of freedom [6]. This work is related to ongoing research in quantum information processing, and it has been pointed out that the damaging effects of decoherence and of probing the system makes the analysis more complicated, and that one may benefit from incorporating error-correction schemes, developed in the context of quantum computation [7]. Recently, an entirely different approach was proposed [8]: rather than requiring complete control of the state of all the atoms, one assumes an optically thick gas with very many atoms N, and one considers complete absorption of non-classical light beams by this gas. Instead of actively squeezing the atomic spin, this is a process in which properties of the incident field are *mapped* onto the collective atomic variables. In relation to a proposed experimental implementation, the absorption of a coherent field on one transition $|0\rangle \leftrightarrow |1\rangle$ and absorption of a squeezed field on another transition $|0\rangle \leftrightarrow |2\rangle$ in V-type atoms was analyzed in detail in [8]. The atoms decay by spontaneous emission back to the ground state $|0\rangle$ but the atomic populations in the excited states (much smaller than N) and coherences restricted to the excited state manifold reflect the statistics of the fields, and it was shown that a weak probe, coupling the states $|1\rangle$ and $|2\rangle$ to a further excited level, will be sensitive to the atomic fluctuations, and that measurements may benefit from their reduction compared to those of a classically excited gas.

In this paper, we shall consider a simpler variant of the proposal, in which there is no need to consider the spatio-temporal character of the light propagation problem as in [8].

In Section 2, we consider the absorption of twin beams of light, where the photons in each pair excite two different transitions in the atoms. We investigate the fluctuations in the population difference of the excited atomic states when only a fraction of the light beams is absorbed. In Section 3, we turn to ground states, where we propose a method which can lead to near-vanishing population fluctuations by quantum non-demolition measurements and feed-back optical pumping. In the concluding Section 4, we discuss the role of band-width of the non-classical radiation fields, and we describe some natural generalizations of the work presented.

2 Atomic twin-correlations induced by absorption of twin-beams of light

We consider a gas of atoms with one lower and two upper states, illuminated by a light field consisting of twin-correlated photons. The number of photons in either of the two modes fluctuates, but we have identically the same number of photons in each mode, since the photons are assumed generated in a down-conversion process, *e.g.*, in a non-degenerate optical parametric oscillator (OPO).

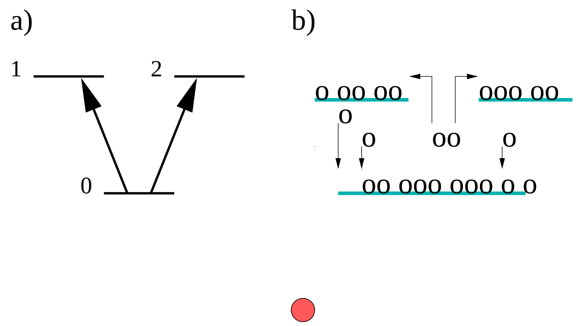

Fig. 1. (a) V-atoms excited by incident twin-beams, one field-mode excites the $|0\rangle \rightarrow |1\rangle$ transition, the other excites the $|0\rangle \rightarrow |2\rangle$ transition. (b) Classical analog: balls are put in pairs on two shelves, from which they, independently, may fall back on the floor.

We assume that the two field modes interact with the two optical transitions $|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$ as indicated in Figure 1. In analogy with our proposal in reference [8], we may imagine that the two twin-correlated field modes correspond to photons of opposite circular polarization so that the dipole selection rules ensure perfect association of each field mode with a specific transition between atomic Zeeman sub-levels. We may alternatively imagine different excited levels, so that resonance conditions associate the field modes with the atomic transitions. We shall in the following only need the property that the fields are thus distinguishable by the atoms.

2.1 Complete absorption

If all photons of both beams are absorbed in the gas, *i.e.* if it is optically thick, the difference $(N_1 - N_2)$ in numbers of atoms populating the two excited states does not change due to interaction with the incident radiation field. Note, we have no control over *which* atoms are excited by photons from one or the other field mode, it is the *total* excited state populations over the entire gas that are considered. The atoms decay by spontaneous emission of light. This occurs independently for atoms in states $|1\rangle$ and $|2\rangle$, and will hence lead to fluctuations in the population difference. If the atomic decay rate is Γ, the probability distribution $P(N_1, N_2)$ of having a definite number of atoms in each of the excited states changes due to spontaneous emission. The rate of change of $P(N_1, N_2)$ is given by the equation

$$\begin{aligned}\frac{d}{dt}P(N_1,N_2)|_{loss} = &-\Gamma N_1 P(N_1,N_2) - \Gamma N_2 P(N_1,N_2)\\ &+\Gamma(N_1+1)P(N_1+1,N_2)\\ &+\Gamma(N_2+1)P(N_1,N_2+1) \qquad (1)\end{aligned}$$

where the terms in the first line describe processes in which N_1 and N_2 atoms are excited but one of them decays, and the second and third lines describe the contribution from processes where one atom too many is excited in one of the states, and where a spontaneous emission therefore increases the probability of having a certain set of occupation numbers N_1 and N_2.

From equation (1) one obtains the rate of change of the mean population difference $\langle N_1 - N_2\rangle$ by multiplying on both sides by $(N_1 - N_2)$ and carrying out the summation over all values (the terms of the second and third lines with displaced arguments in the probability distribution are treated by a change of variable $N_i \leftrightarrow N_i + 1$). This yields

$$\frac{d}{dt}\langle N_1 - N_2\rangle|_{loss} = -\Gamma\langle N_1 - N_2\rangle, \qquad (2)$$

and in the same way, we obtain the equation for the rate of change of the variance

$$\frac{d}{dt}\langle (N_1-N_2)^2\rangle|_{loss} = -2\Gamma\langle (N_1-N_2)^2\rangle + \Gamma\langle N_1+N_2\rangle. \qquad (3)$$

In steady state, the equation for the variance is readily solved and shows that

$$\langle (N_1 - N_2)^2\rangle = \langle N_1 + N_2\rangle/2. \qquad (4)$$

If the transitions had been excited by classical fields, N_1 and N_2 would have been independent variables distributed according to Poisson distributions, and the variance would have been twice as large.

The result is not surprising: the atoms are excited in pairs and their subsequent independent decays can only partly (50%) deteriorate this correlation established among different atoms in the gas. The fluctuations end up half-way between the perfectly matching incident fields and the uncorrelated outgoing fluorescence, just like the amplitude fluctuations, considered in [8]. Note also the complete analogy between our treatment of the atomic excitation and the analysis of the intracavity-fields in a non-degenerate OPO: in the OPO the populations of the two intra-cavity field modes are fed by the non-linear crystal, and the linear loss out of the cavity leads to the same relations for the intracavity photon number difference as our equations (2, 3) and to the same factor two reduction in steady state compared to coherent states.

2.2 Incomplete absorption

It was pointed out in reference [8] that complete absorption of the light field is important. This is, in fact, more easily understood and analyzed in case of populations than in the case of coherences: if a light field with a large definite number of photons n is transmitted through a gas, and only a fraction f of the light intensity is absorbed (on the average), the actual number of absorbed photons will be described by a binomial distribution, and in the limit of small f and large n, this will approach a Poisson distribution, indistinguishable from the result of classical irradiation.

We consider now the case of twin beam absorption, where the same intensity fraction f of both fields is absorbed. In addition to the above terms describing the rate of change of the population difference (2, 3), we must now consider the changes occurring due to imperfect absorption.

Assume that with a rate of κ, n photons are incident on the gas in mode 1 and the same number is incident in mode 2. Photon numbers n_1 and n_2 are transmitted, and the non-vanishing fluctuations of $n_1 - n_2$ represent a source of fluctuations of the atomic population difference.

$$\begin{aligned}\frac{d}{dt}\langle N_1 - N_2\rangle|_{trans} &= \kappa\langle(n-n_1)-(n-n_2)\rangle \\ &= \kappa f\langle n - n\rangle = 0. \qquad (5)\end{aligned}$$

On the average, the same number of atoms will be excited into either of the two excited states, but for the variance we obtain the equation

$$\begin{aligned}\frac{d}{dt}\langle (N_1 - N_2)^2\rangle|_{trans} &= \kappa(Var(n_1)+Var(n_2)) \\ &= \kappa f(1-f)\langle n + n\rangle, \qquad (6)\end{aligned}$$

where the last result is due to the number of transmitted photons being binomially distributed, given the incident number of photons in either mode.

The total number of excited atoms $N_1 + N_2$ on the average equals the absorbed photon flux, divided by the atomic decay rate, $\Gamma\langle N_1+N_2\rangle = f\kappa\langle n+n\rangle$, hence we can replace the photon numbers by atomic excitation numbers in equation (6), add this equation to equation (3), and obtain the steady state variance

$$\langle (N_1 - N_2)^2\rangle = \left(\frac{2-f}{2}\right)\langle N_1 + N_2\rangle. \qquad (7)$$

This equation interpolates between the 50% reduction compared to classical noise for $f = 1$ and the classical result when $f \to 0$.

The same results are obtained if we replace the physical situation by one of complete absorption of fields, which before entering the gas, have been passed though a beam splitter with a transmission probability of f.

3 Ground state twin atoms

It is difficult to imagine how to reduce the number fluctuations in atomic excited states further, since in steady state the linear uncorrelated loss processes exactly balance the correlated absorptions. It thus seems natural to try to correlate the populations in atomic ground states (or meta-stable states) in order to reduce the devastating effects of spontaneous decay.

3.1 Super-binomial distribution by complete absorption

We first try out the idea of complete absorption, and we consider, *e.g.*, a gas of Λ-atoms, in which absorption from twin beams of light may serve to equilibrate the two

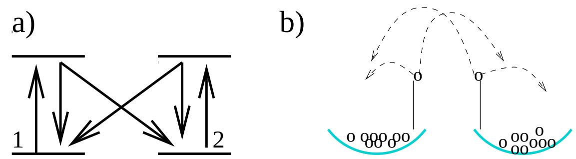

Fig. 2. (a) Atomic level scheme in which atoms excited out of two ground states $|1\rangle$ and $|2\rangle$ by optical beams may decay back into their initial state or into the other ground state. (b) Classical analog: balls from two buckets are thrown into the air and fall back into the buckets at random.

ground state populations. In order to eliminate the potential complication of stimulated Raman-processes, we consider atoms with two lower and two upper states, so that excitations occur from the ground states to different excited states, from which spontaneous decay may take the atoms back into the initial state or into the other ground state. This situation is illustrated in Figure 2.

The sketched process is indeed able to set up correlations between the atoms. If the branching ratios from decay of the upper states are identical for both ground states, the probability for any atom to be in either ground state is one half in the long time limit, but the populations N_1 and N_2 do not follow a binomial distribution: if at some point we have N_1 and N_2 atoms in the ground states, the absorption of a photon pair followed by spontaneous emission events, with a probability of 1/2 leaves the atomic populations unchanged and with identical probabilities 1/4 an atom is transferred from state $|1\rangle$ to $|2\rangle$ or from $|2\rangle$ to $|1\rangle$. As a consequence, in steady state all values of N_1 get the same probability, and the fluctuations of $N_1 - N_2$ are much *larger* than for a binomial distribution on the two states.

In fact, one obtains the same result if uncorrelated fields with same mean intensity are absorbed: with probability 1/2 the next photon is in mode 1, thus with probability 1/4 an atom may be transferred from state $|1\rangle$ to $|2\rangle$, with probability 1/4 an atom may be transferred from state $|2\rangle$ to $|1\rangle$, and with the remaining probability of 1/2 the populations are unchanged.

It is the assumption of complete absorption of the incident photons, independently of the atomic populations, that causes the disappearance of the "usual" combinatorial dependence on N_1 and N_2. If the medium is not optically thick, an overweight of atoms in one ground state will cause stronger absorption from the corresponding field mode, and hence the populations will be equilibrated approaching the binomial distribution in case of only weak absorption.

3.2 Feed-back optical pumping

We shall now incorporate another idea, which has also been applied in the preparation of non-classical light: feedback [9,10].

Our proposal is indeed very simple. Rather than absorbing fields on the two transitions, described above, we imagine that they are used for a QND detection of the

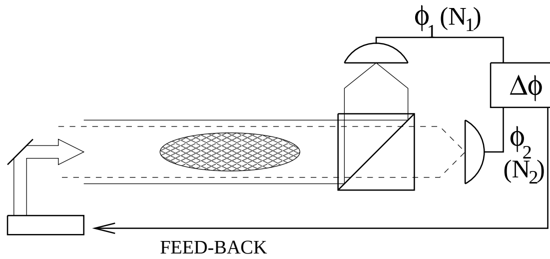

Fig. 3. Feed-back loop. Off-resonant probe fields, sensitive to the number of atoms in each of two ground states, are detected. The reading of their phase-difference $\phi_1 - \phi_2$ is fed back to a laser which emits a resonant light pulse which causes a specified average amount of optical pumping in the atomic gas.

numbers of atoms in the respective ground states. We consider for example the dispersive interaction with off-resonant probes causing phase-shifts of the fields proportional to the number of atoms [11]. In an interferometric set-up one may directly measure the phase-difference of two probes corresponding to a measurement of the population difference $N_1 - N_2$.

Given a measurement of $N_1 - N_2$, we know the population difference exactly, thus the fluctuations have already been eliminated, but let us assume, that we want to maintain a vanishing number-difference, *e.g.*, in a situation where a weak external perturbation may cause changes in the populations. In this case, we act back on the atomic sample in accordance with the measurement result of the QND detection. If we detect a population difference of $N_1 - N_2 = k_0$ we either inject a pulse with mean photon number k_0 being completely absorbed, or a stronger pulse with a detuning, ensuring that *on average* k_0 atoms are excited. In both implementations, the pulse should excite on average k_0 atoms out of ground state $|1\rangle$ if $k_0 > 0$ and $|k_0|$ atoms out of state $|2\rangle$ if $k_0 < 0$.

A set-up for this suggestion is illustrated in Figure 3.

Although the idea of complete absorption is not compatible with the following calculation, it is instructive to apply an idealized version of this scheme to the situation of only two atoms, being initially distributed on the internal states according to a binomial distribution. If, the atoms are found in the same state, they are both excited (by complete absorption of two photons), and with a probability of 1/2 they decay into different states, and the desired result is obtained, whereas with probabilities of 1/4 they both decay into one or the other ground state. This means that the desired state with $N_1 = N_2$ has a probability, which is updated as $p \to p + (1-p)/2 = (p+1)/2$, and $\langle (N_1 - N_2)^2 \rangle$ is reduced by a factor of two in each iteration.

In Figure 4 we present calculations, based on a similar up-dating of the probability distribution for 400, 800 and 1200 atoms. The figure shows the variance as a function of the number of applications of coherent pulses with mean photon number equal to the detected population difference. In the relevant limit of many atoms, we find a very rapid reduction of the fluctuations. According to our procedure, when we detect a value for N_1, we apply a coherent pulse of photons after which the mean populations are equal, but the variance of N_1 equals the variance of the number of excited atoms returning to either of the ground states. According to the Poisson distribution, corresponding to the binomial splitting of a coherent pulse, this variance equals its mean, *i.e.*, $\mathrm{Var}(N_1)_{after} = |N_1 - N/2|$. Averaged over the initial probability distribution of N_1, $P(N_1)$, the variance of the distribution after the first iteration is thus

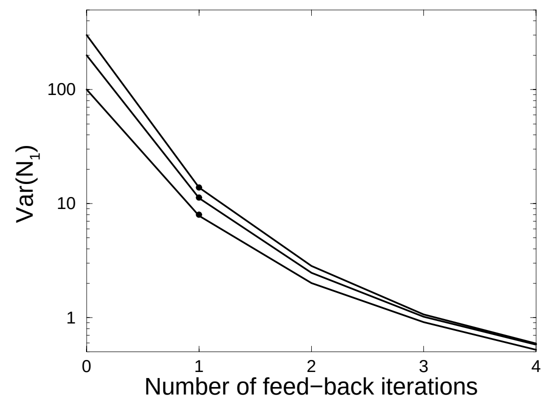

Fig. 4. Variance of the population of one atomic ground state as a function of the number of feed-back optical pumping events, (lines are drawn to guide the eye). From below, the curves correspond to initial binomial distributions of 400, 800 and 1200 atoms. Filled circles represent the approximation (9), valid after one feed-back iteration.

$$\mathrm{Var}(N_1)_{after\ feed-back} = \sum_{N_1} P(N_1)|N_1 - N/2|. \qquad (8)$$

This quite unusual "first moment" of the initial distribution can be easily computed, and for a Gaussian distribution, which offers an excellent approximation to the binomial distribution of many particles, it leads to the variance reduction after the first feed-back optical pumping pulse has been applied:

$$\mathrm{Var}(N_1)_{after\ feed-back} = \sqrt{2\mathrm{Var}(N_1)/\pi}. \qquad (9)$$

The variance in atom numbers is reduced to its square-root, as confirmed by the numerical results in Figure 4.

Equation (9) is the main result of this section, applicable for a distribution which is initially well-described by a Gaussian.

As long as the variance remains large compared to unity, which is by the way not very long, we may analytically investigate the effect of subsequent iterations, and one obtains that they continue to reduce the variance by extracting the square-root and multiplying by a numerical factor of order unity involving Gamma-functions of argument $(1/2 + 1/2^n)$, where n is the number of iterations.

If a sub-Poissonian pulse is applied, the variance is reduced by yet another factor between unity and two (it is two for the absorption of a number state pulse due to the binomial splitting of the decaying atoms).

4 Discussion

Like in the case of excited state atomic spin squeezing, it is worth emphasizing, that it is not a definite part of the atoms that are in one or the other ground state. Neither is the sample in a coherent superposition of states like the so-called Dicke states [12]. This is effectively prevented by the principal possibility to monitor the decay and in this way identify the randomly selected set of atoms populating one and the other state. Still, we may imagine probing schemes, sensitive to effects accumulated over the whole sample, where the reduced fluctuations, discussed in this paper may be useful.

We wish to draw attention to an aspect concerning the optical twin-correlations, that was neglected in Section 2. The atomic sample absorbs and re-emits photons, and the atomic excitation derived from each photon has a lifetime of Γ^{-1} in the system. Within this time scale we must assume perfect twin-correlations of the incident field modes, *i.e.*, identical photon numbers. Now, over a short time interval, the photon numbers may not be perfectly matched since one photon of a pair may leave the OPO cavity before or after its partner. It is thus seen, that the photon lifetimes in the OPO cavity must be shorter than the atomic excited state lifetime, to ensure that photon twins are mapped on atomic twins, residing simultaneously in the excited atomic states in the sample. This is actually equivalent to the requirement, identified in [8], that the spectrum of non-classical light must be broader than the atomic transition. Formalism exists to treat the temporal aspects of feed-back correctly [9,10], and we are currently investigating to which extent this formalism may be adapted to our suggestion.

In case of ground state twins we do not see a similar principal problem, since the atomic states are stable. In a practical experiment, however, one may settle with a finite resolution ΔN of the QND probing, and this will then set the limit for the obtainable width of the final population distribution. This emphasizes that even in a feed-back scheme, you do not get something for nothing [9,10]: we must have sub-binomial resolution to get a sub-binomial distribution.

From a practical perspective, of course, if we are not able to resolve the atomic fluctuations to this precision, we shall not be sensitive to the corresponding noise in our precision measurement either – the ability to produce twin atoms comes with the need for them! We may also state our goals in a different manner: given that squeezing and other quantum correlation phenomena may be realized in optical fields, we investigate in this work whether we can map, transfer or copy some of these phenomena onto collective atomic variables. The squeezing of light that may improve our optical phase measurement and eventually our atom number detection is thus "transformed" into a number squeezing in the atomic sample.

We have considered only the reduction of fluctuations in occupation numbers. We are presently considering other collective atomic operators, relevant to atomic ground states with larger Zeeman degeneracy, involving *e.g.*, the 9 different states of the $J_g = 4$ ground state in Cs. We imagine that with QND probing of different circular or linear polarization components of fields, followed by suitable feed-back optical pumping, a variety of classes of fluctuations may be addressed and modified.

Let us finally mention the possibility to apply the probing and feed-back optical pumping on different and spatially separate samples of atoms. Our scheme, in principle allows us to ensure that two traps contain very closely the same number of atoms in specific internal states, and it thus offers improved sensitivity to local external perturbations of the samples changing these numbers. Our work is thus also related to a recent proposal for improved sensitivity of atomic interferometers by use of QND measurements of atomic populations [13].

We recall that the scheme works in an entirely classical manner, *e.g.* describable by Bertlmann's socks [14] or by balls in buckets, like in Fig. 2b); further generalization, involving *quantum* entanglement of remote samples, are currently under investigation [15].

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